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Normalisation of the background independent open string field theory action

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Abstract: It has been shown recently that the background independent open string field theory provides an exact description of the tachyon condensation on unstable D-branes of bosonic string theory. In this analysis the overall normalisation of the action was chosen so that it reproduces the conjectured relations involving tachyon condensation. In this paper we fix this normalisation by comparing the on-shell three tachyon amplitude computed from the background independent open string field theory with the same amplitude computed from the cubic open string field theory, which in turn agrees with the result of the first quantised theory. We find that this normalisation factor is in precise agreement with the one required for verifying the conjectured properties of the tachyon potential.

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The 26-dimensional bosonic string theory contains Dp-branes for all p. Each of these Dp-branes has a tachyonic mode. It has been conjectured [1] that there is a local minimum of the tachyon potential which describes the closed string vacuum without any D-brane. At this minimum the negative contribution from the tachyon potential exactly cancels the tension of the D-brane. Further, it has been conjectured that a codimension q lump solution on the Dp-brane represents a D$(p-q)$-brane in the same theory. Support for these conjectures comes from the analysis of the world-sheet theory [2, 1], cubic open string field theory (COSFiT) [3], noncommutative limit of the effective field theory of the tachyon [4], as well as various toy models of tachyon condensation [5, 6].

A different open string field theory that is (formally) background independent was proposed and developed in refs. [7]–[11]. Recently it has been pointed out [12, 13] that this string field theory can provide an exact verification of these conjectures. A general field configuration in background independent open string field theory (BIOSFiT) is associated with a boundary operator of ghost number 1 in the world-sheet field theory of matter and ghost system. We shall take the world sheet to be a disc of unit radius with flat metric on it and work in the convention $\alpha' = 1$. If $\{\mathcal{O}_I\}$ denotes a complete set of boundary vertex operators of ghost number 1, we can expand a general operator $\mathcal{O}$ of ghost number 1 as

$$\mathcal{O} = \sum_I \lambda^I \mathcal{O}_I \,. \tag{1}$$

We shall restrict to operators of the form $\mathcal{O} = c\mathcal{V} = \sum_\alpha \lambda^\alpha c\mathcal{V}_\alpha$, where c is the ghost field and $\mathcal{V} = \sum_\alpha \lambda^\alpha \mathcal{V}_\alpha$ is a boundary operator in the matter theory. In this case, a string field theory configuration associated with the operator $c\mathcal{V}$ is described by the world-sheet action

$$\mathcal{S}_{\text{Bulk}} + \int_0^{2\pi} \frac{d\theta}{2\pi} \mathcal{V}(\theta)\,, \tag{2}$$

where the angle θ parameterises the boundary of the disc, and $\mathcal{S}_{\text{Bulk}}$ denotes the bulk world-sheet action corresponding to the closed string background. We shall consider a trivial background in flat space, therefore $\mathcal{S}_{\text{Bulk}}$ describes the CFT of 26 free scalar fields X^μ and the (b, c) ghost system. For such configurations, the string field theory action $S_{BI}(\lambda^\alpha)$ is obtained as a solution of the equation:

$$\frac{\delta S_{BI}}{\delta \lambda^\alpha} = \frac{K}{2} \int \frac{d\theta}{2\pi} \int \frac{d\theta'}{2\pi} \left\langle \mathcal{O}_\alpha(\theta) \{Q_B, \mathcal{O}(\theta')\} \right\rangle_{\mathcal{V}} \,, \tag{3}$$

where $\langle \cdots \rangle_{\mathcal{V}}$ denotes correlation function in the world-sheet field theory described by the action (2). Q_B is the BRST charge and K is a normalisation constant to be fixed later. In the following, we shall also use the correlation function $\langle \cdots \rangle$ in the absence of the boundary term in (2). Notice that eq. (3) determines the action upto an additive constant. However, since we shall always compute the difference

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between the values of the action for two configurations, this ambiguity will not affect our analysis. The subscript BI in eq. (3) stands for background independent open string field theory.

A special class of string field configurations corresponding to operators of the form $\mathcal{O} = c\mathcal{V}$ with:

$$\mathcal{V} = a + \sum_i u^i (X^i)^2 \,, \tag{4}$$

was analysed in refs. [8, 13]. In this case the action can be computed *exactly* since the worldsheet theory remains free. The resulting action has an unstable extremum at $(a = 0, u^i = 0)$ corresponding to the original D-brane. In addition, it has several other extrema with the following properties:

1. There is an extremum at $(a = \infty, u^i = 0)$. The difference in energy density between the original configuration $(a = 0, u^i = 0)$ and this extremum is K [13].[1]

   Thus *if* $K = T_p$ — the tension of the original Dp brane — this would *prove* the conjecture that the tachyon potential has an extremum where the negative contribution due to the potential energy exactly cancels the tension of the D-brane.

   We shall refer to the solution $(a = \infty, u^i = 0)$ as the *vacuum* solution.

2. There is a solution where

   $$u^i = \begin{cases} \infty & \text{for } 1 \leq i \leq q \,, \\ 0 & \text{otherwise} \,, \end{cases} \tag{5}$$

   and a determined as a function of the u^i's [13]. This configuration describes a codimension q soliton with energy per unit $(p - q)$-volume, measured above the energy of the *vacuum solution*, given by:

   $$\Delta\mathcal{E} = (2\pi)^q K \,. \tag{6}$$

   If $K = T_p$, this is precisely the correct formula for the tension of the D$(p - q)$-brane.

It is clear from above that in order to establish that the tachyon dynamics in the background independent open string field theory reproduces the conjectured relations involving tachyon condensation, we need to show that $K = T_p$. This is what we shall prove in this paper. To this end, we compute the on-shell three tachyon amplitude from the cubic open string field theory [14], and compare it with the same amplitude

[1] Throughout this paper we shall be using the convention that when $\mathcal{V} = 0$, the partition function of the matter conformal theory on the unit disk is equal to the volume Ω of the D-brane world-volume. With this convention, $S_{BI}(a, u^i = 0) = \Omega K (1 + a) e^{-a} + C$ where C is an additive constant. This differs somewhat from the convention used in ref. [13].

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computed in the background independent string field theory [7]. (Notice that the three point tachyon amplitude calculated in ref. [13] vanishes on-shell. This does not agree with the result of the first quantised theory.)

Recall that the euclidean action of the cubic open string field theory describing the dynamics of a Dp-brane is given by

$$S_{\text{cubic}} = 2\pi^2 T_p \left(\frac{1}{2}\langle \Phi | Q_B | \Phi \rangle + \frac{1}{3}\langle f_1 \circ \Phi(0) f_2 \circ \Phi(0) f_3 \circ \Phi(0) \rangle \right), \tag{7}$$

where $|\Phi\rangle$ is the string field represented by a ghost number 1 state in the Hilbert space of the first quantised theory, f_i's are known conformal maps reviewed in ref. [15], and $f_i \circ \Phi(0)$ denotes the conformal transform of the vertex operator $\Phi(0)$ by f_i. The normalisation factor $2\pi^2 T_p$ was derived in ref. [15], and will be important for our analysis. The Fock vacuum $|k\rangle \equiv e^{ik\cdot X(0)}|0\rangle$, where $|0\rangle$ is the $\mathrm{SL}(2,\mathbb{R})$ invariant vacuum and k^μ labels momentum along the world-volume of the D-brane, is normalised as follows:

$$\langle k | c_{-1} c_0 c_1 | k' \rangle = (2\pi)^{p+1} \delta(k + k') \,. \tag{8}$$

We shall interpret $(2\pi)^{p+1}\delta(0)$ as the volume of the Dp-brane world-volume.

Let us consider a tachyonic string field configuration of the form

$$|\Phi\rangle = \int d^{p+1}k \, T(k) c_1 |k\rangle \,, \tag{9}$$

with $T(k)$ supported over near on-shell momentum $k^2 \simeq 1$. Substituting (9) into (7), and keeping only the leading order terms in $(k^2 - 1)$ in both the quadratic and the cubic terms, we arrive at the action:

$$\begin{aligned} S_{\text{cubic}} \simeq 2\pi^2 T_p \Bigg[&\frac{1}{2} \int d^{p+1}k \int d^{p+1}k' \, (2\pi)^{p+1} \delta(k + k')(k^2 - 1) T(k) T(k') + \\ &+ \frac{1}{3} \int d^{p+1}k \int d^{p+1}k' \int d^{p+1}k'' (2\pi)^{p+1} \delta(k + k' + k'') T(k) T(k') T(k'') \Bigg]. \end{aligned} \tag{10}$$

This encodes information about the on-shell three tachyon amplitude in the cubic open string field theory.

Let us now turn to the background independent open string field theory. In this case a near on-shell tachyon field configuration is represented by a boundary perturbation of the form:

$$\int \frac{d\theta}{2\pi} \mathcal{V}(\theta) = \int \frac{d\theta}{2\pi} \int d^{p+1}k \, \phi(k) e^{ik\cdot X(\theta)} \,, \tag{11}$$

with $\phi(k)$ supported over near on-shell momentum $k^2 \simeq 1$. We shall use the normalisation (8) for computing correlation functions in the world-sheet theory in the absence of any boundary perturbation. The action of the background independent open string field theory is given by eq. (3). To calculate the quadratic term it is sufficient to replace $\langle\ \rangle_{\mathcal{V}}$ by $\langle\ \rangle$ and use the relation

$$\{Q_B, c(\theta)e^{ik\cdot X(\theta)}\} = (k^2-1)\partial c(\theta)c(\theta)e^{ik\cdot X(\theta)}\,. \tag{12}$$

Substituting this in eq. (3), and keeping terms to leading order in (k^2-1), we get the near on-shell quadratic term:

$$S^{(2)}_{BI} = \frac{K}{4}\int d^{p+1}k \int d^{p+1}k'\,(2\pi)^{p+1}\delta(k+k')(k^2-1)\phi(k)\phi(k')\,. \tag{13}$$

Next we evaluate the on-shell three tachyon coupling. Unfortunately, direct determination of this coupling is difficult due to the problem with ultraviolet divergences on the world-sheet [11], so we shall take recourse to an indirect method.[2] We use the fact that whenever the world-sheet action (2) describes a conformal field theory, the corresponding string field configuration is a solution of the equations of motion [7]. Thus the equations of motion derived from the string field theory action S_{BI} must be proportional to the β-functions of the boundary conformal field theory described by the action (2). Now if,

$$\mathcal{V} = \sum_\alpha \lambda^\alpha \mathcal{V}_\alpha\,, \tag{14}$$

where $\mathcal{V}_\alpha$ are primary vertex operators of dimensions $h_\alpha \simeq 1$, with the operator product expansion

$$\mathcal{V}_\alpha(x)\mathcal{V}_\beta(y) \simeq \frac{C^\gamma_{\alpha\beta}}{|x-y|^{h_\alpha+h_\beta-h_\gamma}}\mathcal{V}_\gamma(y)\,, \tag{15}$$

the β-function associated with the coupling λ^α to second order in λ is given by [16]

$$\beta^\alpha(\lambda) \propto (h_\alpha-1)\lambda^\alpha + \frac{1}{2\pi}C^\alpha_{\beta\gamma}\lambda^\beta\lambda^\gamma\,. \tag{16}$$

The factor of $(2\pi)^{-1}$ in front of the second term can be traced to the normalisation factor of $(2\pi)^{-1}$ appearing in front of the boundary perturbation in eq. (2). Since the operators $e^{ik\cdot X}$ have conformal weights $h(k)=k^2$, and satisfy the operator product expansion

$$e^{ik\cdot X(x)}e^{ik'\cdot X(y)} \simeq \frac{1}{|x-y|^{k^2+k'^2-(k+k')^2}}e^{i(k+k')\cdot X(y)}\,, \tag{17}$$

the equation of motion for the (near on-shell) tachyon field in BIOSFiT is:

$$(k^2-1)\phi(k) + \frac{1}{2\pi}\int d^{p+1}k' \int d^{p+1}k''\,\delta(k-k'-k'')\phi(k')\phi(k'') = 0\,. \tag{18}$$

[2] Renormalization group equations have been used in the past to derive on- and off-shell tachyon amplitudes in open string theory [17].

The cubic part of the background independent string field theory action can now be constructed, the normalisation being determined from eq. (13). Near on-shell the BIOSFiT action upto cubic order in $\phi(k)$ is given by,

$$S_{BI} \simeq \frac{K}{4}\Big[\int d^{p+1}k \int d^{p+1}k'\,(2\pi)^{p+1}\delta(k+k')(k^2-1)\phi(k)\phi(k') + \\ + \frac{1}{3\pi}\int d^{p+1}k \int d^{p+1}k' \int d^{p+1}k''\,(2\pi)^{p+1}\delta(k+k'+k'')\phi(k)\phi(k')\phi(k'')\Big]. \tag{19}$$

The above can be compared with the corresponding result of the cubic open string field theory (10). The quadratic terms in the two actions agree under the identification:

$$T(k) = \frac{1}{2\pi}\sqrt{\frac{K}{T_p}}\,\phi(k) + \cdots\,, \tag{20}$$

where the dots denote terms linear in $\phi(k)$ which vanish on-shell, and terms quadratic and higher orders in $\phi(k)$. This relates the tachyon fields in the two string field theories. Requiring that the cubic terms match leads to

$$K = T_p\,, \tag{21}$$

the relation we set out to prove. Thus the background independent string field theory provides a verification of all the conjectures involving tachyon condensation on the bosonic D-branes. It should be possible to generalise this to prove the corresponding conjectures in superstring theory.

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